Labor-market Volatility in a Matching Model with Worker Heterogeneity and Endogenous Separations

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Abstract

Recessions are times when the quality of the unemployment pool is lower, because entry into unemployment is biased in favor of low-productivity workers. I develop a search and matching model with worker heterogeneity and endogenous separations that has this feature. I show that in a recession a compositional shift in unemployment towards low-productivity workers, due an increase in job separations, lowers the matching effectiveness of searching firms, thereby causing their average recruiting cost to rise. This acts to further depress vacancy creation in a recession. In contrast to most models that allow for endogenous separations, this model generates a realistic Beveridge curve correlation.

JEL classification: E24; E32; J63; J64

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Evidence clearly suggest that when the overall unemployment rate rises, it rises by much more for workers with lower productivity. For example, using education as a proxy for productivity, Nickell and Bell (1995) show that ratio of the “low-education” unemployment rate to the “high-education” unemployment rate in the US rose from 3.1 in the early 1970s to 4.5 in the early 1980s.\textsuperscript{1} Evidence on the separation rates for workers in different education categories also indicate that the jobs that are lost in downturns are mainly lower-skill jobs, since entry into unemployment in downturns is biased in favor of lower skills.\textsuperscript{2} In a recession there are therefore a lot more unemployed workers looking for jobs, but the quality of the unemployment pool is lower. Although this is a well known fact, in the vast majority of the literature on search and matching models workers are assumed to be ex-ante homogenous. Thus, such cyclical changes in the quality of the unemployment pool have largely been overlooked.

In this paper, I build a matching model that has this feature. Workers are assumed to be ex-ante heterogeneous in terms of their inherent ability or human capital and job separations occur endogenously, as in Mortensen and Pissarides (1994). Specifically, in a recession firms become more selective about the productivity of the jobs they wish to operate, resulting in a spike in job destruction. The workers that are laid off in downturns are the least productive. For this reason, recessions are not only associated with an increase in unemployment, but also a deterioration of the quality of the unemployment pool. I then show that allowing for this feature adds new insights into the cyclical behavior of unemployment and vacancies in matching models; an issue that has received considerable attention in the recent literature, due to the work of Shimer (2005) who argued that the textbook matching model with a constant job separation rate fails to generate sufficient volatility in key labor market variables.\textsuperscript{3}

It is common for models with volatility in job separations to generate, counterfactually, a positively sloped Beveridge curve, reflecting the endogenous response of vacancies to an increase in unemployment caused by an increase in job separations. In the present model, by contrast, fluctuations driven by changes in job separations generate a negatively sloped Beveridge curve, that is more consistent with the data, because they generate not only
significant counter-cyclical fluctuations in unemployment but also significant pro-cyclical fluctuations in market tightness (the vacancy to unemployment ratio). The mechanism for amplification works through the impact of changes in the quality of the unemployment pool on the average recruiting cost of searching firms. The workers that are laid off in downturns are those whose productivity falls below the firms’ acceptance threshold, i.e., those whose productivity is no longer high enough to be worthwhile for firms to keep them. These workers congest the market during downturns making it more difficult for firms to locate workers whose productivity is above their acceptance threshold. This raises the average recruiting cost of firms, and therefore acts to further depress vacancy creation in a recession. The rise in unemployment among low-productivity workers that occurs during a recession will facilitate vacancy creation only when aggregate productivity improves, and thus firms become less selective about the productivity of the matches they wish to form. Hence, in the model developed in this paper, fluctuations in job separations, induce counter-cyclical fluctuations in the average recruiting cost of firms, thereby amplifying the response of not only unemployment but also vacancies.

Evidence that some of the newly unemployed workers fail to quickly find jobs, suggest that the amplification mechanism this paper emphasizes is empirically relevant. Coles and Petrongolo (2003) find that only about half of the newly unemployed workers manage to quickly find jobs, the rest remain unemployed and their re-employment rate depends statistically on the inflow of new vacancies instead of the stock of vacancies. This suggest that some of the newly unemployed workers have skills in relatively short demand. These workers are therefore less likely to find a job from the current stock of vacancies, and more likely to exit unemployment when economic conditions improve and new vacancies come into the market; consistent with the set up developed in this paper. Likewise, Shimer (2008) finds that the re-employment probability declines with unemployment duration, leading to the coexistence of a large number of short unemployment spells with a small number of workers who stay unemployed for much longer. This too is consistent with the setting developed in this paper.

As Rogerson and Shimer (2010), among others, show, recessions are not only characterized by a prolonged decline in the job finding rate, but also by a sharp and short-lived
increase in the job separation rate, suggesting that the matching model needs have both of these features in order to adequately explain labor market volatility. Specifically, whenever cyclical changes in the separation rate are excluded, the model misses on the share of variation in the unemployment rate due to its inflow rate. However, as shown in Mortensen and Nagypál (2007b), the most widely used matching model that features such spikes in job destruction, due to Moretensen and Pissarides (1994) (the MP model, henceforth), delivers much more volatility in the unemployment rate, but no more volatility in market tightness than a model with constant separation rate. This implies that vacancies and unemployment must covary positively, yielding an upward-sloping Beveridge curve, which is at odds with the data.

This failure is to be found in the assumption that a worker’s productivity follows an exogenous process that has no memory at the individual level, which implies that a spike in job destruction has no impact on the composition of the unemployment pool in terms of productivity. Instead, it just raises the number of unemployed, thereby raising the arrival rate or searching workers to vacant firms, thus lowering average recruiting costs, and in turn, encouraging firms to open more vacancies. The current model’s main departure from the MP model is that it allows for match productivity to depend on worker ability instead of just being randomly drawn. For this reason, in the current model a spike in job destruction causes a compositional shift towards marginal workers, which, in contrast to the MP model, causes the matching effectiveness of searching firms to fall, instead of rise.

Pries (2008) and Bils et al. (2010) have also recently emphasized a new avenue for cyclical volatility in vacancy creation that reflects compositional changes in the unemployment pool. They argue that a compositional shift towards workers who generate a smaller surplus to employers can further discourage vacancy creation in a recession. The former allows for heterogeneity in workers’ productivity, and imposes the shift exogenously by considering an exogenous rise in the separation rate of low-productivity workers. There seems to be no theoretical justification for such an exogenous change; but more importantly, since Pries’s model does not feature endogenous separations, i.e., endogenous changes in firms’ acceptance threshold, it does not feature counter-cyclical changes in average recruiting costs.
either. The model in Bills et al., features heterogeneity in workers’ reservation wages, and predicts that in recessions separations become skewed towards workers with higher reservation wages who generate smaller expected surplus to employers. The findings of both papers suggest that these compositional shifts generate more volatility in vacancy creation, but substantial amount of volatility remains to be explained by other factors. My paper contributes to this literature by showing that the decline in vacancy creation in a recession may also reflect a compositional shift that raises average the average recruiting cost of searching firms by lowering their matching effectiveness.

The rest of the paper is organized as follows. Section I, lays out the set up of the model under study. Section II characterizes the steady state equilibrium. Section III presents comparative static results that characterize the response of key labor-market variables to aggregate productivity shocks. Section IV presents some quantitative results. Section V discusses some generalizations of the model and Section VI concludes.

I THE MODEL

The economy is populated by ex-ante heterogeneous risk-neutral workers of measure one and firms of a large measure. Workers differ in terms of their inherent and observable ability or human capital, which is measured by $x$. The allocation of ability across workers is assumed to be uniformly distributed over $[0,1]: x \sim F[0,1]$. In any period a worker may be either matched with a firm or unemployed, while a firm may be either matched with a worker and producing or unmatched and posting a vacancy. A type-$x$ worker produces $y_t p(x)$ units of output, where $y$ is aggregate productivity and $p(x)$ is worker-specific productivity, which is increasing in the worker’s ability: $p'(x) > 0$. Unemployed workers receive a constant flow benefit of $b$ per period. Firms that post a vacancy pay a constant cost of $c$ per period.

Let $u_t(x)$ and $v_t$ denote the number of unemployed workers of type $x$ and posted vacancies, respectively, in period $t$. The total number of contacts between searching workers and firms in period $t$ is determined by a matching function $m(v_t, u_t) = v_t^{1-\alpha} u_t^\alpha$, where $u_t = \int_0^1 u_t(x)dx$ gives the total number of unemployed workers in period $t$. The
probability that a worker contacts a firm can be written as \( m(\theta_t) \), where \( \theta_t = \frac{\nu_t}{u_t} \) measures the tightness of the labor market. Likewise, a vacancy is contacted by a worker (of any type) with probability \( q(\theta_t) \) and by a worker of type \( x \) with probability \( q(\theta_t) \frac{a_t(x)}{u_t} \).

Once a searching worker and firm meet, they negotiate on a contract that divides the surplus of the match between the worker and the firm in fixed proportions in line with Nash Bargaining. The worker’s bargaining weight is \( \beta \) and the disagreement point is the severance of the match. Let \( S_t(x) \) denote the surplus of a match with a worker of type \( x \) in period \( t \) and let \( U_t(x), W_t(x), J_t(x) \) and \( V_t \) be the values received by an unemployed and an employed type-\( x \) worker, the value of a match with a type-\( x \) worker to the firm, and the value of a vacancy-posting firm, respectively. The match surplus is defined as 
\[ S_t(x) = W_t(x) + J_t(x) - U_t(x) - V_t(x) \]
and the wage rate, \( w_t(x) \), satisfies the Nash solution, 
\[ W_t(x) - U_t(x) = \beta S_t(x). \]

The worker and the firm will agree to continue the match only if \( S_t(x) \geq 0 \). They will choose to separate if separation is jointly optimal, in which case \( S_t(x) < 0 \). Since the surplus of a match is increasing in the worker’s productivity the firm and the worker will choose to adopt a reservation policy: they will choose to continue or form any match with 
\[ p(x) \geq p(R_t), \]
where \( p(R_t) \) is the reservation productivity, and \( R_t \) the reservation ability. Aside from the jointly optimal separations, known as endogenous separations, matches also face a risk of separating for exogenous reasons with a probability \( s \).

**Timing**

The timing within a period is as follows. Aggregate productivity, \( y_t \), is determined at the beginning of the period, before production takes place. Matches with \( x < R_t \) are destroyed, and the rest produce. After production takes place, a randomly selected fraction \( s \) of matches break up for exogenous reasons, firms post vacancies to ensure zero profits, and search takes place.
Value Functions

The values of unemployment and unemployment for a worker of ability \( x \), solve

\[
U_t(x) = b + \gamma E_t \left[ m(\theta_t) \max\{W_{t+1}(x), U_{t+1}(x)\} + (1 - m(\theta_t))U_{t+1}(x)\right]
\]

\[
W_t(x) = w_t(x) + \gamma E_t \left[ sU_{t+1}(x) + (1 - s)W_{t+1}(x)\right]
\]

where \( E_t \) is the expectation operator and \( \gamma = \frac{1}{1+r} \) is the discount factor. The value of a match with a worker of type \( x \) to a firm is given by

\[
J_t(x) = y_t p(x) - w_t(x) + \gamma E_t \left[ sV_{t+1}(x) + (1 - s)J_{t+1}(x)\right]
\]

Finally, the value of a vacancy satisfies:

\[
V_t = -c + \gamma E_t \left[ q(\theta_t) \int_0^1 \frac{u_t(x)}{u_t} \max\{J_{t+1}(x), V_{t+1}\} dx + (1 - q(\theta_t))V_{t+1}\right]
\]

The interpretation of the value functions is straightforward. In (1) the payoff in the current period for an unemployed worker is \( b \). With probability \( m(\theta_t) \) the worker finds a vacancy, yielding an employment value \( W_{t+1}(x) \). The worker will get hired only if \( W_{t+1}(x) \geq U_{t+1} \), which, given the wage determination process, implies \( J_{t+1}(x) - V_{t+1} \geq 0 \) so that the firm is willing to hire him. Otherwise, the worker remains unemployed yielding a value \( U_{t+1} \). With probability \( 1-m(\theta_t) \) the worker does not find a vacancy and the continuation value is \( U_{t+1} \). In (2), an employed worker earns the wage \( w_t(x) \), keeps the job to the next period with probability \( 1 - s \) and loses it with probability \( s \). In (3), a filled job produces output \( y_t p(x) \) and pays the wage \( w_t(x) \). With probability \( 1 - s \) the match continues to the next period and with probability \( s \) the match separates and the job becomes vacant. Finally, in (4), a firm with a vacancy incurs a cost \( c \) and meets with a type-\( x \) worker with probability \( q(\theta_t) \frac{u_t(x)}{u_t} \). Matching with this worker yields a value \( J_{t+1}(x) \). The firm will therefore choose to hire the worker only if \( J_{t+1}(x) \geq V_{t+1} \), otherwise the firm remains vacant yielding a value \( V_{t+1} \). With probability \( 1 - q(\theta_t) \) the firm does not meet a worker and thus remains unfilled yielding a value \( V_{t+1} \).

Unemployment

The law of motion for unemployment is

\[
u_{t+1}(x) = u_t(x) + s (f(x) - u_t(x)) - u_t(x)m(\theta_t)I + \delta(x)
\]
\( \delta(x) \) captures discrete jumps from employment to unemployment due to endogenous separations, and \( I \) is an indicator function, which takes the value of 1 if the worker’s ability is equal or above the reservation ability and 0 otherwise. More precisely: \( \delta(x) = 0 \) and \( I = 1 \), if \( x \geq R_t \), but \( \delta(x) = (1 - s) (f(x) - u_t(x)) \) and \( I = 0 \), otherwise.

**Equilibrium**

Using the the Nash solution, the value in (4) can be written as:

\[
V_t = -c + \gamma E_t \left[ (1 - \beta)q(\theta_t) \int_0^1 \frac{u_t(x)}{u_t} S_{t+1}(x)dx + V_{t+1} \right]
\]

In a free-entry equilibrium \( V_t = 0 \) holds for all \( t \); thus \( \theta_t \) is determined by:

\[
\frac{c}{q(\theta_t)} = \gamma (1 - \beta) E_t \int_0^1 \frac{u_t(x)}{u_t} S_{t+1}(x)dx
\]

Using the free-entry condition, the Nash solution, and the value functions in (1) to (3) we can write the surplus of a match with a worker of ability \( x \) as:

\[
S_t(x) = \max\{y p(x) - b + \gamma E_t S_{t+1}(x) [1 - s - \beta m(\theta)] , 0\}
\]

Equations (5) to (7) determine the equilibrium path of \( \theta_t \) for given realizations of the aggregate state and distribution of unemployment across worker types.

## II Steady State

Next, I characterize the steady state equilibrium where the distribution of workers across unemployment and the aggregate state \( y \) is constant.

In a steady-state equilibrium there exists a cutoff productivity \( p(R) \) such that \( S(x) \geq 0 \) if and only if \( p(x) \geq p(R) \). To understand why notice that the steady-state surplus of a match with a worker of type \( x \) satisfies

\[
S(x) = \max\{\frac{y p(x) - b}{\gamma (r + s + \beta m(\theta))} , 0\}
\]

Hence, \( S(x) \geq 0 \) if and only if \( p(x) \geq p(R) \), and

\[
p(R) = \frac{b}{y}
\]
determines the reservation ability, $R$.

The steady-state distribution of workers across unemployment is given by:

$$ u(x) = \frac{s f(x)}{s + m(\theta)} \quad (10) $$

if $x \geq R$, and

$$ u(x) = f(x) \quad (11) $$

otherwise. The overall unemployment rate can be calculated as $\int_0^R f(x)dx + \int^1_R \frac{sf(x)}{s+m(\theta)}dx$, which gives:

$$ u = \frac{s + F(R)m(\theta)}{s + m(\theta)} \quad (12) $$

It is clear from (9) that $R$ is decreasing in $y$, meaning $F(R)$ is also decreasing in $y$. Hence, counter-cyclical movements in the reservation productivity, and thus ability, induce counter-cyclical movements in the unemployment rate.

Workers with $x < R$ separate at rate one, while workers with $x \geq R$ separate at rate $s$. The overall separation rate can therefore be written as

$$ \tilde{s} = F(R) + (1 - F(R))s \quad (13) $$

The average job finding and job filling rates differ from the contact rates $m(\theta)$ and $q(\theta)$, respectively, because only workers whose productivity is above the reservation productivity can find a job. In particular, the average job finding and filling rates can be calculated as $\tilde{m} = \int_R^1 \frac{u(x)}{u} m(\theta)$ and $\tilde{q} = \int_R^1 \frac{u(x)}{u} q(\theta)$, which give:

$$ \tilde{m} = \phi(R, \theta)m(\theta) $$
$$ \tilde{q} = \phi(R, \theta)q(\theta) \quad (14) $$

where

$$ \phi(R, \theta) = \frac{s(1 - F(R))}{s + F(R)m(\theta)} \quad (15) $$

The free-entry condition that determines the steady-state value of $\theta$ is,

$$ \frac{c}{q(\theta)} = \gamma(1 - \beta) \int_R^1 \frac{u(x)}{u} S(x)dx \quad (16) $$

With (8), (10) and (11) substituted in, the free-entry condition can be written as

$$ \frac{c}{q(\theta)\phi(R, \theta)} = \frac{(1 - \beta)(y\bar{p} - b)}{r + s + \beta m(\theta)} \quad (17) $$
where \( \tilde{p} = \frac{\int x p(x) dF(x)}{1 - F(R)} \) is the average worker-specific productivity among the employed. The free-entry condition is such that the expected surplus from filling a vacancy (right-hand-side) equals the average recruiting cost (left-hand-side). If the expected surplus is higher than the average recruiting cost then firms create more vacancies per job seeker until all rents are exhausted.

The main difference between this model and other models that allow for countercyclical movements in job separations is the presence of \( \phi(R, \theta) \) in the free-entry condition. A large \( \phi(R, \theta) \) means that the average recruiting cost is lower, because the probability a firm will encounter a worker whose productivity is higher than the reservation productivity is higher. Notice from (15) that \( \phi(R, \theta) \) is decreasing in \( R \), meaning that a rise in the reservation productivity (and thus ability) raises the average recruiting cost. This, coupled with the fact that the reservation productivity is decreasing in \( y \), implies that countercyclical movements in the reservation productivity induce countercyclical movements in the average recruiting cost, thereby amplifying the response of vacancies to aggregate productivity shocks. Unlike other models, in this model a rise in unemployment due a spike in job destruction does not facilitate the creation of new jobs. This is because the workers that are laid off in downturns are those whose productivity is no longer high enough to be worthwhile for the firms to hire them. These workers congests the market making more difficult for firms to locate workers whose productivity is above the reservation productivity, as captured by the negative relation between \( R \) and \( \phi(R, \theta) \). The presence of \( \phi(R, \theta) \) in the free-entry condition therefore captures an additional source of fluctuations in vacancy creation, which is absent from existing models with volatility in job separations.

For the results below, it is also useful to characterize the replacement ratio, i.e., the ratio of the opportunity cost of employment to the worker to average labor productivity. The replacement ratio in the model is \( \tilde{b} = \frac{b_y}{\tilde{p}} \).
Next, I derive comparative static results that describe how the key labor-market variables in the model respond to changes in aggregate productivity.\(^5\)

By taking logs of (12) and differentiating the result with respect to \(\ln y\) we obtain the following expression for the elasticity of the unemployment rate with respect to aggregate productivity:

\[
\frac{\partial \ln u}{\partial \ln y} = -\frac{(1 - \alpha)m(\theta)}{s + m(\theta)} \left[ \frac{s(1 - F(R))}{s + F(R)m(\theta)} \right] \frac{\partial \ln \theta}{\partial \ln y} + \frac{m(\theta)f(R)R}{s + F(R)m(\theta)} \frac{\partial \ln R}{\partial \ln y} \tag{18}
\]

The first term captures the effect of changes in aggregate productivity on the unemployment rate through the impact of such changes on market tightness. The second term reflects the changes in the unemployment rate due to counter-cyclical movements in the reservation productivity. Clearly, the negative response of the reservation productivity amplifies the response of unemployment to aggregate productivity shocks. Thus, unlike the canonical model with a constant separation rate (canonical model, henceforth), analyzed in Shimer (2005), the model extended to include endogenous separations has the potential to explain the volatility of unemployment.

The model can generate a larger volatility of unemployment than the canonical model, but if the increased volatility of unemployment is not accompanied by an increase in the volatility of market tightness, the model will fail to account for the Beveridge curve. Specifically, if the response of market tightness in the present model remains as small as in the canonical model, then the larger negative response of the unemployment rate will be accompanied by a counter-cyclical response in the vacancy rate, because

\[
\frac{\partial \ln v}{\partial \ln y} = \frac{\partial \ln \theta}{\partial \ln y} + \frac{\partial \ln u}{\partial \ln y} \tag{19}
\]

If \(\frac{\partial \ln v}{\partial \ln y}\) is large, while \(\frac{\partial \ln u}{\partial \ln y}\) remains small, then the former dominates the latter so that \(\frac{\partial \ln v}{\partial \ln y}\) is, countefactually, negative.

Substituting (18) into (19) gives

\[
\frac{\partial \ln v}{\partial \ln y} = \left[ 1 - \frac{(1 - \alpha)m(\theta)}{s + m(\theta)} \right] \frac{s(1 - F(R))}{s + F(R)m(\theta)} \frac{\partial \ln \theta}{\partial \ln y} + \frac{m(\theta)f(R)R}{s + F(R)m(\theta)} \frac{\partial \ln R}{\partial \ln y} \tag{20}
\]

As can be seen, a large positive response in market tightness implies a large positive response in the vacancy rate. But the negative response of the reservation productivity
dampens the response of the vacancy rate, as captured by the second term in the above expression. This means that the model can explain jointly the behavior of unemployment and vacancies, only if in addition to the larger volatility in unemployment it generates a sufficiently larger volatility in market tightness than the canonical model.

The MP model, which, as mentioned earlier, is the most popular model of endogenous separations, predicts a counter-cyclical vacancy rate, because it does not deliver more volatility in market tightens than the canonical model. The elasticity of tightens in the canonical model with only a constant separation rate \(s\) and identical workers that produce \(p\) is given by

\[
\left(\frac{\partial \ln \theta}{\partial \ln y}\right) = \frac{r + s + \beta m(\theta)}{\alpha(r + s) + \beta m(\theta)} \left[\frac{1}{1 - \tilde{b}^c}\right]
\]

(21)

where \(\tilde{b}^c = \frac{b}{yp}\) gives the replacement ratio. As shown in Mortensen and Nagypál (2007b), the elasticity of market tightness in the MP model is observationally equivalent to that in the canonical model, given in (21). Thus, when both models are calibrated in the same way, i.e., given equal replacement ratios, average job finding and separation rates, and parameter values for \(\alpha, r\) and \(\beta\), they yield identical elasticities of market tightness.

The current model has the potential to deliver more volatility in market tightness than both the canonical and the MP model, because, as explained above, counter-cyclical movements in the reservation productivity induce counter-cyclical movements in vacancy creation costs. By taking logs of the free-entry condition in (17) and differentiating the result with respect to \(\ln y\) we obtain the following expression for the elasticity of market tightness:

\[
\left(\frac{\partial \ln \theta}{\partial \ln y}\right) = \frac{r + s + \beta m(\theta)}{\alpha(r + s) + \beta m(\theta)} \left[\frac{\partial \ln \phi(R, \theta)}{\partial \ln y} + \frac{1}{1 - b}\right]
\]

(22)

Notice that the elasticity in (22) differs from the one in (21) by the term \(\frac{\partial \ln \phi(R, \theta)}{\partial \ln y}\), which reflects the impact of changes in aggregate productivity on the probability a searching firm will locate a worker suited for its vacancy, i.e., a worker whose productivity is above the reservation productivity. If this term is positive then the elasticity of tightness with respect to aggregate productivity in the present model can be higher than that in both the MP and canonical model.
By taking logs of (15) and differentiating with respect to \( \ln y \) we obtain

\[
\frac{\partial \ln \phi(R, \theta)}{\partial \ln y} = -\frac{(1 - \alpha) F(R)m(\theta)}{s + F(R)m(\theta)} \frac{\partial \ln \theta}{\partial \ln y} + \frac{m(\theta)f(R)R}{s + F(R)m(\theta)} \frac{\partial \ln R}{\partial \ln y} \] (23)

The first term reflects the fact that a fall in tightness, and as a consequence, a fall in the job finding rate, due to a fall in aggregate productivity, raises unemployment among workers whose productivity falls above the reservation productivity, thereby making it easier for firms to fill their vacancies. The second term captures the impact of counter-cyclical movements in the reservation productivity. In a recession the reservation productivity rises, resulting in spike in job destruction. The workers who enter unemployment are those whose productivity falls below the reservation productivity. These workers congest the market making it more difficult for firms to locate workers whose productivity is above the reservation productivity. As shown in Section IV, for reasonable calibrations this second effect dominates so that \( \frac{\partial \ln \phi(R, \theta)}{\partial \ln y} > 0 \), and the elasticity of market tightness in this model is higher than in both the MP and the canonical model.

The elasticity of the job finding rate, \( \hat{m} \), with respect to aggregate productivity can be expressed as:

\[
\frac{\partial \ln \hat{m}}{\partial \ln y} = \left[ \frac{s(1 - \alpha)}{s + F(R)m(\theta)} \right] \frac{\partial \ln \theta}{\partial \ln y} - \frac{f(R)R}{1 - F(R)} \left[ \frac{s + m(\theta)}{s + F(R)m(\theta)} \right] \frac{\partial \ln R}{\partial \ln y} \] (24)

The job finding rate responds to changes in aggregate productivity due to the impact of such changes on both the market tightness (first term) and the reservation productivity (second term).

The separation rate responds to aggregate productivity shocks due to the impact of such changes on the reservation ability. Specifically,

\[
\frac{\partial \ln \hat{s}}{\partial \ln y} = \frac{f(R)R(1 - s)}{\hat{s}} \frac{\partial \ln R}{\partial \ln y} \] (25)

Finally, by taking logs of the job destruction condition in (9) and differentiating with respect to \( \ln y \), we can write the elasticity of the reservation ability with respect to aggregate productivity as:

\[
\frac{\partial \ln R}{\partial \ln y} = -\frac{1}{\epsilon_p(R)} \] (26)

where \( \epsilon_p(R) \) denotes the elasticity of the productivity function \( p(x) \) with respect to \( x \), evaluated at \( x = R \). Notice that if this elasticity is large then the response of the reservation ability to aggregate productivity shocks is small. Intuitively, if small differences
in workers' abilities imply large differences in their productivities, then a large change in aggregate and thus reservation productivity implies only a small change in the reservation ability.

From equations (25) and (26) it is clear that the response of the separation and thus unemployment rate to aggregate productivity shocks depends on two things. First, on how productivity is distributed among workers with different ability; i.e, the exact functional form of $p(x)$, which determines $\epsilon_p(x)$; and second, on how ability is allocated across workers; i.e., the distribution $F(x)$. If there are large differences in the productivities of workers with different abilities, or if the share of workers whose ability is marginal is small, then a large increase in the reservation productivity does not necessarily imply a large increase in unemployment. In contrast, in the MP model a large increase in the reservation productivity implies also a large increase in the unemployment rate, because all workers are identical and thus are uniformly affected.

This difference turns out to be important for how the elasticities of the separation and unemployment rates respond to a change in the replacement ratio. From equations (21) and (22) it is evident that a higher replacement ratio in our calibrations implies a larger elasticity of market tightness. In fact, as Hadedorn and Monovskii (2008) have argued, if the replacement ratio is high, about 95% of market returns, then the canonical model can calibrate the observed cyclical volatility in market tightness. However, as Mortensen and Nagypál (2007b) show, once we allow for endogenous separations, the increased volatility of tightness due to a small increase in the replacement ratio comes at the cost of an excessively volatile separation and unemployment rate, and in turn, a strongly countercyclical vacancy rate. A higher replacement ratio reduces the firm’s steady state profits so that cyclical shocks have a bigger proportional impact on profits, and thus job creation. But it also implies that cyclical shocks have a larger proportional impact on reservation productivity, thereby making the separation rate in the MP model excessively volatile. Since in the current model a large response in the reservation productivity does not necessarily translate into a large response in the separation rate, as I demonstrate in Section IV, the model yields reasonable amounts of variation in both the market tightness and the separation rate at empirically plausible values for the replacement ratio.
It is also worth mentioning here that an additional channel through which the model can generate a larger change in the vacancy rate relative to that in labor productivity is the divergence between aggregate and labor productivity, which is a common feature of models that allow for endogenous separations. A percentage increase in the aggregate component of productivity, $y$, translates into a smaller percentage increase in average labor productivity, $\tilde{y} \tilde{p}$, because an increase in aggregate productivity lowers the reservation productivity, thereby lowering average worker-specific productivity, $\tilde{p}$. Specifically,

$$\frac{\partial \ln \tilde{y} \tilde{p}}{\partial \ln y} = 1 + \left[ \frac{f(R) R}{1 - F(R)} + \frac{b R}{y \int_{R}^{1} p(x) f(x) dx} \right] \frac{\partial \ln R}{\partial \ln y} \quad (27)$$

Apparently, $\frac{\partial \ln \tilde{y} \tilde{p}}{\partial \ln y}$ is less than one, because $\frac{\partial \ln R}{\partial \ln y}$ is negative. When confronting the model with the data, the appropriate measure of the changes in a variable $z$ relative to the changes in labor productivity is given by

$$\frac{\Delta_y \ln z}{\Delta_y \ln \tilde{y} \tilde{p}} \equiv \frac{\partial \ln z}{\partial \ln \tilde{y} \tilde{p}} \frac{\partial \ln y}{\partial \ln \tilde{y} \tilde{p}} \quad (28)$$

This means that the change in variable $z$ relative to that in labor productivity is larger in this model than that in the canonical model. Moreover, since both models are calibrated to match the empirical volatility of the average productivity of labor, $y$-shocks are larger in this model than in the canonical model. In turn, larger $y$-shocks generate larger fluctuations in the key labor market variables.

### IV Some Quantitative Results

Next, I present quantitative results of the model. In my baseline calculations I use the parameter values and targets used by Shimer (2005) to facilitate direct comparability of my results with the results of Shimer’s canonical model and the MP model analyzed by Mortensen and Nagypál (2007b), using also the same parameter values and targets. Hence, aggregate productivity is normalized to $y = 1$ and the quarterly discount rate is $r = 0.012$. I set the elasticity parameter to $\alpha = 0.72$, and let worker’s bargaining power take the same value, $\beta = 0.72$. The value of leisure, $b$, the exogenous separation rate, $s$, and the vacancy cost $c$, are set so that: 1) the implied average quarterly separation
rate (endogenous and exogenous together) is 0.10, 2) the implied job finding rate is 1.355, and 3) the replacement ratio equals 0.40. I choose the number for $\epsilon_p(R)$ that implies an unemployment elasticity that is equal to the empirical elasticity of $-3.88$ and examine how the results change if we allow different values for $R$.

Table 1 reports the model-implied elasticities of the key labor-market variables both with respect to aggregate and with respect to labor productivity. I use the notation $\epsilon_{i,j}$ to denote the elasticity of the variable $i$ with respect to variable $j$. The table also reports the results of the canonical model and the relevant empirical responses (labeled as data) based on Tables 3 and 1, respectively, in Shimer (2005). The separations elasticity that matches the empirical elasticity of unemployment is very close to that in the data. The resulting elasticity of tightness is much larger than that of the canonical model and much closer to that in the data. The current model clearly outperforms the canonical model when it comes to explaining the volatility of tightness. Therefore, unlike the MP model, this model predicts a procyclical vacancy rate. However, as can be seen, it still under-predicts the volatility of the vacancy rate.

Table 1: Model results at $\tilde{b} = 0.4$

<table>
<thead>
<tr>
<th>$F(R)$</th>
<th>$\epsilon_{\theta,y}$</th>
<th>$\epsilon_{v,y}$</th>
<th>$\epsilon_{s,y}$</th>
<th>$\epsilon_{m,y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5%</td>
<td>4.15</td>
<td>0.66</td>
<td>-1.54</td>
<td>3.71</td>
</tr>
<tr>
<td>1%</td>
<td>4.14</td>
<td>0.66</td>
<td>-1.54</td>
<td>3.71</td>
</tr>
<tr>
<td>2%</td>
<td>4.12</td>
<td>0.63</td>
<td>-1.50</td>
<td>3.70</td>
</tr>
<tr>
<td>3%</td>
<td>4.09</td>
<td>0.59</td>
<td>-1.46</td>
<td>3.69</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$F(R)$</th>
<th>$\epsilon_{\theta,g\tilde{p}}$</th>
<th>$\epsilon_{v,g\tilde{p}}$</th>
<th>$\epsilon_{s,g\tilde{p}}$</th>
<th>$\epsilon_{m,g\tilde{p}}$</th>
<th>$\epsilon_{y,g\tilde{p}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5%</td>
<td>4.64</td>
<td>0.76</td>
<td>-1.75</td>
<td>4.15</td>
<td>0.90</td>
</tr>
<tr>
<td>1%</td>
<td>4.62</td>
<td>0.74</td>
<td>-1.72</td>
<td>4.14</td>
<td>0.90</td>
</tr>
<tr>
<td>2%</td>
<td>4.58</td>
<td>0.69</td>
<td>-1.67</td>
<td>4.11</td>
<td>0.90</td>
</tr>
<tr>
<td>3%</td>
<td>4.53</td>
<td>0.65</td>
<td>-1.62</td>
<td>4.09</td>
<td>0.90</td>
</tr>
<tr>
<td>data</td>
<td>7.56</td>
<td>3.68</td>
<td>-1.97</td>
<td>2.34</td>
<td>1.72</td>
</tr>
<tr>
<td>canonical</td>
<td>1.72</td>
<td>1.35</td>
<td>0.50</td>
<td>0.90</td>
<td>0.90</td>
</tr>
</tbody>
</table>

16
Table 2: Model results at $\tilde{b} = 0.71$

<table>
<thead>
<tr>
<th>$F(R)$</th>
<th>$\epsilon_{\theta,y}$</th>
<th>$\epsilon_{v,y}$</th>
<th>$\epsilon_{s,y}$</th>
<th>$\epsilon_{m,y}$</th>
</tr>
</thead>
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<tr>
<td>0.5%</td>
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<td>2.08</td>
<td>-1.46</td>
<td>3.96</td>
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<tr>
<td>1%</td>
<td>5.75</td>
<td>2.05</td>
<td>-1.46</td>
<td>3.95</td>
</tr>
<tr>
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<td>5.70</td>
<td>2.01</td>
<td>-1.47</td>
<td>3.92</td>
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<tr>
<td>3%</td>
<td>5.65</td>
<td>1.95</td>
<td>-1.47</td>
<td>3.89</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$F(R)$</th>
<th>$\epsilon_{\theta,g\bar{p}}$</th>
<th>$\epsilon_{v,g\bar{p}}$</th>
<th>$\epsilon_{s,g\bar{p}}$</th>
<th>$\epsilon_{m,g\bar{p}}$</th>
<th>$\epsilon_{g\bar{p},y}$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>2.18</td>
<td>-1.53</td>
<td>4.15</td>
<td>0.95</td>
</tr>
<tr>
<td>1%</td>
<td>6.04</td>
<td>2.16</td>
<td>-1.53</td>
<td>4.14</td>
<td>0.95</td>
</tr>
<tr>
<td>2%</td>
<td>5.98</td>
<td>2.10</td>
<td>-1.54</td>
<td>4.11</td>
<td>0.95</td>
</tr>
<tr>
<td>3%</td>
<td>5.94</td>
<td>2.05</td>
<td>-1.54</td>
<td>4.09</td>
<td>0.95</td>
</tr>
</tbody>
</table>

| data   | 7.56                        | 3.68                     | -1.97                    | 2.34                     |
| canonical | 1.72                      | 1.35                     | 0.50                     |

Allowing for a higher replacement ratio than Shimer’s 0.4, improves the volatility of the vacancy rate considerably. Shimer (2005) sets the replacement ratio to 0.4, which as Hagedorn and Manovskii (2008) argue, is too low, because it does not allow for the value of leisure or home production. They suggest a replacement ratio of 0.955, which seems implausibly large and $\beta = 0.052$, which seems implausibly small. Mortensen and Nagypál (2007a) show that using these values and letting the rest of the parameters take the values used by Shimer yields an elasticity of market tightness with respect to labor productivity in the canonical model of 26.83, which is implausibly large. Hall (2006) also argues that Shimer’s replacement ratio is too low, but suggest a more reasonable value of 0.71, based on empirical literature on household consumption and demand.

Table 2 shows the results when Hall’s suggestion is used and the value of $\epsilon_{p(R)}$ is again such that the model-implied unemployment elasticity matches the one in the data. This variation improves the volatilities of market tightness and vacancy rate considerably, while keeping the volatility of the separation rate very close to that in the data. In contrast, in the MP model, an increase in the replacement ratio improves the elasticity of market
tightness with respect to labor productivity significantly, but this improvement comes at the cost a much higher elasticity of the separation rate and thus unemployment rate to changes in aggregate productivity. Specifically, as shown in Mortensen and Nagypál (2007b), an increase in the replacement ratio from 0.4 to 0.55 doubles the elasticity of tightness, but the implied separations elasticity is more than 10 times the empirical one, and the implied elasticity of the unemployment is more than 5 times the empirical one. As explained above, this occurs because in the MP model a rise in the reservation productivity affects all workers uniformly, while in the present model it only affects those whose productivity is marginal.

V Discussion

The approach taken in this paper has been to keep the model simple in order to highlight the effects of the changing composition of unemployment on matching effectiveness and in turn on vacancy creation. However, it is worth commenting briefly on some generalizations.

While match productivity has been assumed to depend only on the worker’s ability, it is possible to also allow for a stochastic match-specific component of productivity, as in the MP model. In such a setting, the productivity of a match would depend not only on the worker’s ability, but also on a match-specific productivity draw from a common distribution. As in the current setting, low-ability workers would be less productive and thus generally closer to the separation threshold, meaning that matches with low-ability workers would be less likely to survive during bad times. In a recession the composition of unemployed would therefore shift towards the least productive workers, who are less likely to receive a match-specific productivity draw that is high enough to render their overall productivity above the acceptance threshold. This would cause the average recruiting cost for firms to rise, consistent with the setting described above.

Allowing for match-specific productivity shocks would complicate the model considerably, but would also give a more appealing reason as to why the laid-off workers remain in the unemployment pool in a recession. In the simpler setting described in this paper, I
assume that laid-off workers remain in the unemployment pool, despite being aware that
they will not be able to find a job, unless aggregate economic conditions improve. The
underlying assumption is that these workers search, because they want to be entitled to
unemployment benefits. However, other approaches, as the one suggested above, could
perhaps make the model more realistic. Specifically, with match-specific productivity
shocks all unemployed workers have an incentive to search, because they can receive a
match-specific productivity draw that renders their overall productivity above the accep-
tance threshold.

An other alternative generalization of the present model is to assume that there are
two types of matches: a type-$x$ worker produces $yp(x)$ in a good match, but only $\mu yp(x)$
in a bad match, and $\mu < 1$. There is a threshold productivity on the bad matches so
that firms accept bad matches only for workers of ability $x \geq R$, and $R$ is lower at
higher $y$, but all good matches are acceptable at all $y$. Under these assumptions, in a
recession the threshold productivity rises so that many low-ability workers are laid off, but
have an incentive to stay in the unemployment pool, because they can find a good match,
which is always acceptable. This generalization is also consistent with the current model’s
assumptions. In fact, if the fraction of good matches is very small, then this alternative
setting approaches the setting developed in this paper, in the sense that the its predicted
volatilities would be very close to those of the current model, reported above.

It is worth mentioning, however, that such approaches could make the model more
realistic, but at the same time much more complicated, while adding little to the paper’s
main message: with a match acceptance decision being tighter in recessions and looser
in booms, as it is the case in models with endogenous separations, an important channel
through which worker heterogeneity can increase volatility in vacancy creation is the
changing composition of the unemployment pool that lowers matching effectiveness in
recessions and vice-versa in booms.
VI Conclusion

There is broad consensus that entry into unemployment in a recession is biased in favor of lower skills, so that the quality of unemployment is lower. In this paper I have built a matching model with endogenous separations and worker heterogeneity, that features such cyclical changes in the quality of the unemployment pool.

A nice property of the model is that it generates a realistic Beveridge curve correlation without introducing complex features relative to the most widely used model of endogenous separations, due to Mortensen and Pissarides (1994). The only difference is that my model allows for match productivity to depend on worker’s ability, which I consider to be a natural assumption, instead of just being randomly drawn. A key feature of the model that follows from this assumption is that a rise in job destruction in a recession causes the matching effectiveness of searching firms to fall, and in turn, their average recruiting cost to rise. This is because it induces a compositional shift in the pool of searching workers towards workers whose productivity is low, in a period when firms are actually more selective about the productivity of the workers they wish to recruit.

My approach in this paper has been to keep the model simple in order to highlight a new mechanism for amplification that works through the impact of changes in the firms’ match acceptance threshold on the composition of unemployment. By keeping the model simple, I have stressed that this mechanism provides a solution to the standard model’s failure to generate a negatively sloped Beveridge curve, but incorporating this mechanism into more generalized settings, can also shed light on some other important questions that remain open.

One such question, posed in Rogerson and Shimer (2010) is whether search theoretic models of the labor market can explain the wedge between the marginal rate of substitution of consumption for leisure and the marginal product of labor, often called the “labor wedge”. In general search frictions act as an adjustment cost which dampens fluctuations in employment. Specifically, in the presence of search frictions, increasing the vacancy to unemployment ratio in response to a positive productivity shock is costly, because doing so reduces the matching effectiveness of each vacancy posting firm. This puts a break on the growth of employment in expansions. For this reason, as shown in Rogerson and
Shimer (2010), in a model with search frictions the labor wedge is positively correlated with employment, but the opposite holds in the data. As this paper has shown, job destruction shocks can cause procyclical changes to matching effectiveness that amplify the volatility of employment. As firms’ acceptance threshold rises in recession and falls in a boom, some workers are constrained from working as much as they would like in a recession and vice-versa in a boom, which provides a potential explanation to the countercyclical labor wedge. Further investigation along this dimension might give new insights into the cyclical behavior of the labor wedge in the presence of search frictions.
Notes

1Likewise, Nagypál (2001) shows that the unemployment rate among workers with less than high school diploma more than tripled between 1973 and 1983, whereas for workers with a college degree, it merely doubled. For further discussion on these evidence, see Pries (2008).

2See, for instance, Farber et al. (1993), Abraham et al. (1997) and Farber (2003).


4Another example of a model with endogenous separations, but a uniform unemployment pool is the one in Guerrieri (2007). Guerrieri introduces heterogeneity in terms of the effort cost workers have to incur at the beginning of a new employment relationship, but assumes that this effort costs is randomly drawn, so that the unemployment pool is uniform, as in the MP model. This might be one of the reasons she finds that this type of heterogeneity does not amplify the response of the unemployment rate and tends to dampen the response of market tightness to productivity.

5I follow a common practice and use comparative static exercises around the non-stochastic state-state to gauge the cyclical response of the model. See for instance, Mortensen and Nagypál (2007a,b) and Pissarides (2009). For a discussion of how good an approximation comparative statics is, see Mortensen and Nagypál (2007b).

6Notice from (13) and (14) that the exogenous separation rate, \( s \), is smaller than the average separation rate, \( \bar{s} \), while the contact rate \( m(\theta) \) is larger than the average job finding rate, \( \bar{m} \). For this reason, when both the canonical and the present model are calibrated to match the average job finding and separation rates, the term in the first bracket of (22) is smaller than that in (21). Still, if the term \( \frac{\partial \ln \phi(R, \theta)}{\partial \ln y} \) is sufficiently large, the elasticity of market tightness in the present model will be higher than in the canonical model.

7This is based on Table 1 in Shimer (2005), which reports the empirical elasticities

8As Mortensen and Nagypál (2007b) point out, the empirical equivalent to the elasticity of \( x \) with respect to change in \( y \) in the model with endogenous separations is the OLS coefficient \( \rho_{xy} \frac{\sigma_x}{\sigma_y} \), where \( \rho_{xy} \) is the correlation between \( \ln x \) and \( \ln y \) and \( \sigma_x \) is the standard deviation of \( \ln x \). Table 1 reports the same OLS coefficients.
References


